

# Semi-Invariant Algorithm for Color to Gray and Back

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## Abstract

This paper proposes a reversible algorithm to convert color images to gray images with keeping chroma and spatial resolution. K. Braun and R. L. de Queiroz proposed a color-to-gray mapping algorithm. The method can almost recover the original color, but it had theoretical problems that both chroma and spatial resolution deteriorated. Our algorithm succeeded to solve those problems by devising a color embedding technique. By replacing a subband of a luminance component by quantized high-pass and chrominance signals, semi-invariant algorithm for chroma and spatial resolution can be realized. Experimental results show that the proposed method can recover vivid color images with keeping spatial resolution from textured gray images.

## Introduction

Digital images can be classified roughly to 24bit color images and 8bit gray images. We have come to tend to treat colorful images by the development of various kinds of devices. However, there is still much demand to treat color images as gray images from the viewpoint of running cost, data quantity, etc.

We can convert a color image into a gray image by linear combination of RGB color elements uniquely. Meanwhile, the inverse problem to find an RGB vector from a luminance value is an ill-posed problem. Therefore, it is impossible theoretically to completely restore a color image from a gray image. For this problem, recently, colorization techniques have been proposed [1]-[4]. Those methods can re-store a color image from a gray image by giving color hints. However, the color of the restored image strongly depends on the color hints given by a user as an initial condition subjectively.

By the way, in 2005, K. Braun and R. L. de Queiroz proposed a sensational method to convert color images to gray images which can be later decoded and converted back to color. The method was based on wavelet transforms and on replacing bandpass subbands by chrominance signals. So, most of the high-frequency components of the luminance signal are lost, and the spatial resolution of the recovered image deteriorates. Furthermore, the gray image converted from the replaced subbands by the inverse wavelet can overflow the 8bit range. This causes a fall of chroma of color recovered image. Figures 1 and 2 show images with color recovery by Ref. [5]. The chroma and spatial resolution decrease.

In this paper, we propose a color-to-gray mapping technique which can improve those problems, that is, our method can recover color images from color embedded gray images with having almost kept chroma and spatial resolution of original color images. In order to keep the spatial resolution of the original color image, we store the high-frequency subband signals in the low bit packet by compressing the range of data. In order to keep the chroma, we store the chrominance components in the high bit packet through  $\beta$ -transform which will be proposed in this paper.

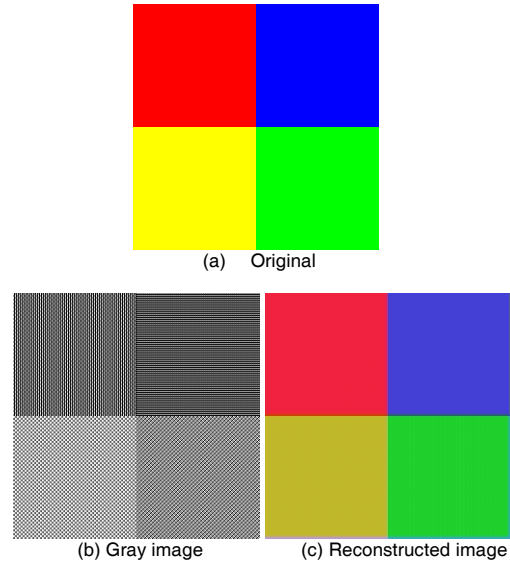


Figure 1 Image "Palette" with color recovery by Ref.[5]

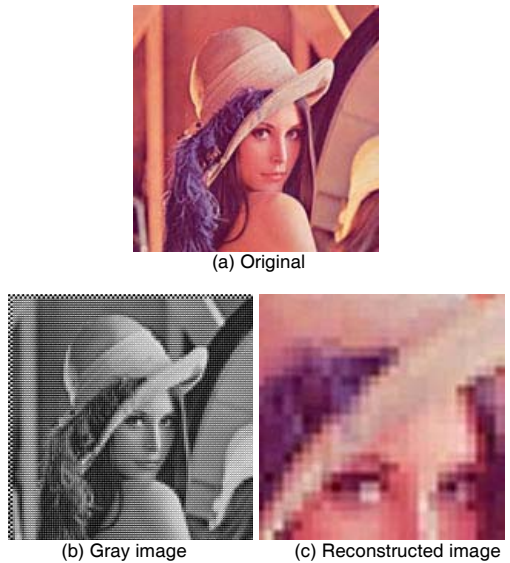


Figure 2 Image "Lena" with color recovery by Ref.[5].

After describing a basic algorithm in Sec. 2, Sec. 3 presents the proposed algorithms to convert color-to-gray images and to recover color information from the textured gray image. Section 4 presents some experimental results. Finally Sec. 5 makes the conclusions of this paper.

## Basic Algorithm

In this section, we describe a basic algorithm for converting color image to grayscale based on the algorithm by K. Braun and R. L. de Queiroz.

E1) Convert images from RGB into  $Y, Cb, Cr$ .

E2) Use a two-level discrete wavelet transform on the luminance  $Y$ , so that the luminance  $Y$  is divided into seven subbands:

$$Y \rightarrow (S_l, S_{h1}, S_{v1}, S_{d1}, S_{h2}, S_{v2}, S_{d2}) \quad (1)$$

E3) Reduce  $Cb$  and  $Cr$  by 1/2, construct  $Cb+, Cb-, Cr+, Cr-$ , and reduce  $Cb-$  further to 1/4 of its original size, where

$$Cb+ = \begin{cases} Cb, & Cb > 0 \\ 0, & Cb \leq 0 \end{cases}, \quad Cb- = \begin{cases} Cb, & Cb < 0 \\ 0, & Cb \geq 0 \end{cases} \quad (2)$$

The same arrangement is made for  $Cr$ .

E4) Replace subbands

$$S_{d1} \leftarrow Cb-, S_{h2} \leftarrow Cr+, S_{v2} \leftarrow Cb+, S_{d2} \leftarrow Cr- \quad (3)$$

E5) Take inverse discrete wavelet transform to obtain the textured gray image, i.e.,

$$(S_l, S_h, S_v, Cb-, Cr+, Cb+, Cr-) \rightarrow Y' \quad (4)$$

E6) Image  $Y'$  is the resulting gray image.

The color recovery steps in Ref.[5] are as follows:

R1) Read the gray textured image.

R2) Use a discrete wavelet transform to convert the gray image into subbands

$$Y' \rightarrow (S_l, S_{h1}, S_{v1}, S_{d1}, S_{h2}, S_{v2}, S_{d2}) \quad (5)$$

R3) Interpolate  $S_{d1}$ , doubling its resolution.

R4) Make  $Cb = |S_{v2} - S_{d1}|$  and  $Cr = |S_{h2} - S_{d2}|$ , and interpolate  $Cb$  and  $Cr$ , doubling their resolutions.

R5) Remove the embedded subbands, i.e., set  $S_{d1} = S_{h2} = S_{v2} = S_{d2} = 0$ , and take the inverse discrete wavelet transform to find  $Y$  as  $(S_l, S_{h1}, S_{v1}, 0, 0, 0, 0) \rightarrow Y$ .

R6) Convert the  $Y, Cb, Cr$  planes back to RGB.

## Semi-invariant Algorithm for Color to Gray and Back

In this section, we propose a new color-to-gray mapping algorithm and color recovery method with keeping chroma and spatial resolution.

### Color-to-gray Step

Figure 3 shows the procedure of the proposed color-to-gray convert method. Our method works as follows:

From E1) to E3), the steps are the same as the basic algorithm described in the previous section. From the step E4), the algorithm changes as follows:

E4') Quantize all subband signals to low  $\alpha$ -bit packet. Quantize chrominance components  $Cb-, Cr+, Cb+, Cr-$  to high  $(8-\alpha)$ -bit packet. After that, we multiply  $\beta$  ( $0 \leq \beta \leq 1$ ) by the quantized chrominance components. This process is called  $\beta$ -transform in the remaining part. Then we obtain replaced subbands  $\{S_{d1}', S_{h2}', S_{v2}', S_{d2}'\}$  from  $\{S_{d1}, S_{h2}, S_{v2}, S_{d2}\}$  and  $\{Cb-, Cr+, Cb+, Cr-\}$ , respectively. Figure 4 shows the concept of the subband replacing process. As shown in Fig.4 (a), the basic

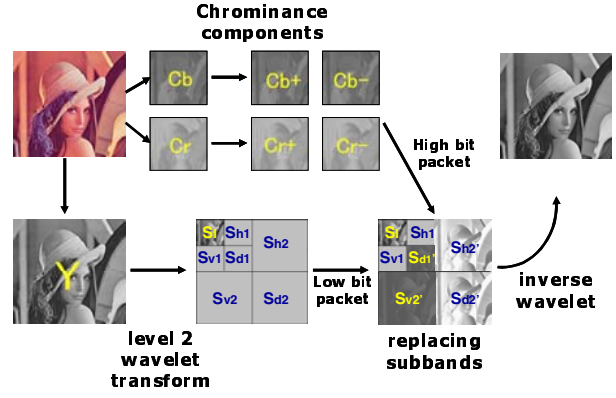


Figure 3 Mapping from color to gray image by the proposed method.

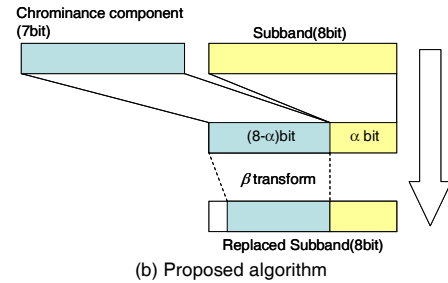
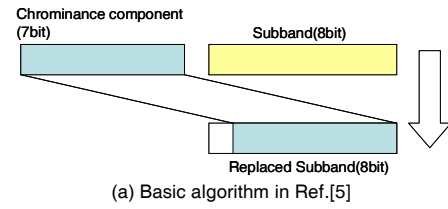


Figure 4 Subband replacing process.

algorithm replaced each subband signal to chrominance components completely. On the other hand, for each subband, both quantized high-frequency signal and chrominance components are stored in the proposed method as shown in Fig.4 (b).

E5') Replace subbands

$$S_{d1} \leftarrow S_{d1}', \quad S_{h2} \leftarrow S_{h2}', \quad S_{v2} \leftarrow S_{v2}', \quad S_{d2} \leftarrow S_{d2}' \quad (6)$$

E6') Take inverse discrete wavelet transform to obtain the textured gray image, i.e.,

$$(S_l, S_{h1}, S_{v1}, S_{d1}', S_{h2}', S_{v2}', S_{d2}') \rightarrow Y' \quad (7)$$

E7') Image  $Y'$  is the resulting gray image.

### Recovery Step

Figure 5 shows the procedure of the proposed color recovery method. Our method works as follows:

R1') Read the gray textured image.

R2') Use a discrete wavelet transform to convert the gray image into subbands

$$Y' \rightarrow (S_l, S_{h1}, S_{v1}, S_{d1}', S_{h2}', S_{v2}', S_{d2}') \quad (8)$$

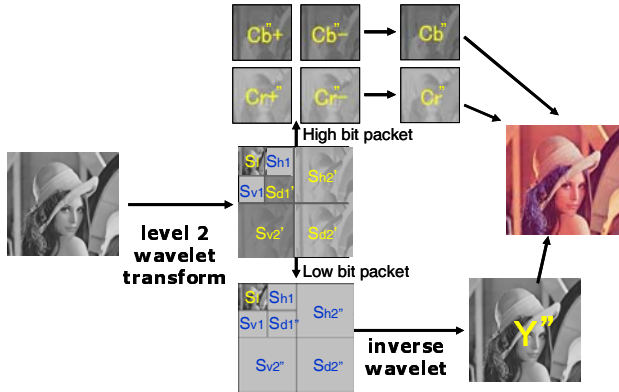


Figure 5 Retrieving color from gray image by the proposed method.

R3') Interpolate  $S_{d1}'$ , doubling its resolution.

R4') Make  $Cb-, Cr+, Cb+, Cr-$  from high bit packet of  $\{S_{d1}', S_{h2}', S_{v2}', S_{d2}'\}$  through  $1/\beta$  transform. Make high-frequency components  $\{S_{d1}'', S_{h2}'', S_{v2}'', S_{d2}''\}$  from low bit packets  $\{S_{d1}', S_{h2}', S_{v2}', S_{d2}'\}$ .

R5') Make  $Cb$  from  $Cb+$  and  $Cb-$ , and  $Cr$  from  $Cr+$  and  $Cr-$ . Then interpolate  $Cb$  and  $Cr$ , doubling their resolutions.

R6') Take the inverse discrete wavelet transform to find  $Y''$  as

$$(S_l, S_{h1}, S_{v1}, S_{d1}'', S_{h2}'', S_{v2}'', S_{d2}'') \rightarrow Y'' \quad (9)$$

R7') Convert the  $Y, Cb, Cr$  planes back to RGB.

The proposed algorithm embeds both chromaticity information and spatial high-frequency components of the luminance signal by using multi-resolutional analysis of wavelet theory. This is the essential idea of the proposed technique. Generally, in natural images, the power of high-frequency subband signal becomes small in comparison with the power of low-frequency components. Therefore we can reduce the number of the quantization bits to express them in  $\alpha$ -bit from 8bit.

### Detailed Algorithm

We describe the details of each step with Figs. 6 and 7. As an example, we consider a case to embed a chrominance component  $Cr+$  to a subband of horizontal direction. Figure 6 (a) shows a decomposition process from four luminance pixels continuing in horizontal direction to  $S_l, S_{h1}, S_{h2}$  by the multi-resolutional analysis in E2).

Figure 6 (b) shows a construction of the replaced subband  $S_{h2}'$  from the original subband  $S_{h2}$  by quantizing high frequency component to  $\alpha$  bit, and embedding  $Cr+$  to  $(8-\alpha)$  bit in steps E4') and E5'). The basic algorithm replaces high-frequency subband signals to chrominance components. So, the resolution of recovered image decreases theoretically. On the other hand, the proposed method maintains high-frequency components. So, the resolution of the recovered image becomes high. Here, there is a trade-off between the spatial resolution of luminance components and the pixel depth of chrominance components, and the parameter  $\beta$  controls the trade-off. Since the effectiveness of the parameter  $\beta$  will be shown in the next subsection, Fig.6 (b) does not perform the  $\beta$ -transform ( $\beta=1$ ). The reason why the chrominance signal is embedded in high bit packet without low bit packet is to obtain

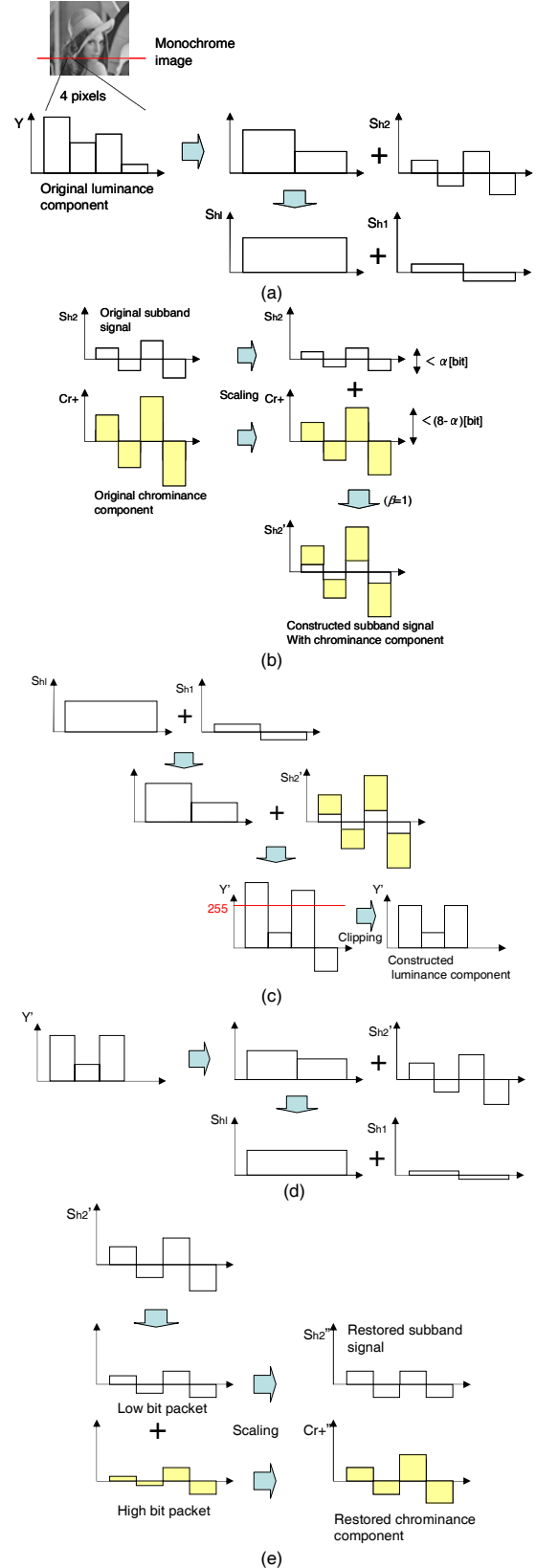


Figure 6 Signal processing without  $\beta$ -transform ( $\beta=1$ ).

almost the same luminance value from the same chrominance value.

Figure 6 (c) shows the construction process of a gray image by the step E6'). In comparison with the original subband signal  $S_{h2}$ , the dynamic range of obtained subband signal  $S_{h2}'$  can be large. So, the luminance value of the constructed gray image can exceed 8bit. Therefore the clipped result becomes the output luminance components of the gray image as shown in Fig.6 (c).

Next, we describe the color recovery processing. Figure 6 (d) shows the decomposed signals  $S_l, S_{h1}, S_{h2}'$  from a luminance signal  $Y'$  by two-level discrete wavelet transform in the step R2'). Here, the subband component  $S_{h2}'$  can become smaller than the original component  $S_{h2}$ , because the luminance signal was clipped. So, as shown in Fig.6 (e), the recovered chrominance component  $Cr+$  through the steps R3') and R4') becomes also smaller than the original component. This will cause a chrominance distortion of the recovered color image. Since the new subband signal  $S_{h2}''$  in Fig.6 (e) was quantized to  $\alpha$ [bit], it seems to cause the distortion of the spatial resolution. But, the influence is small because the power of the most subband components is small.

### Improvement on Chroma Recovery

We explain the effectiveness of the  $\beta$ -transform to improve the distortion of chroma with Fig.7. As shown in Fig.7 (a), the replaced subband  $S_{h2}'$  was constructed by reducing the range of the chrominance component  $Cr+$  with  $\beta$ . By performing the  $\beta$ -transform, most of the luminance value in the constructed gray image exists within the range of 8[bit]. So, the clipping processing becomes needless as shown in Fig.7 (b). In the color recovery steps, we can extract the replaced subband signal  $S_{h2}'$  as shown in Fig.7 (c), and chrominance components are also recovered by  $1/\beta$ -transform without heavy distortion. But there is a trade-off between high chrominance and pixel depth in the  $\beta$ -transform. The parameter  $\beta$  can be fixed for each image, but it is possible to determine the parameter for each image. If we can set the parameter for each subband separately, only users who know the parameter can recover the original color image.

### Experimental Results

In order to verify the performance, we have tested the proposed algorithm. Firstly we show an experimental result for Bride (368x488) in Fig.8 with a zoom-up image around an eye. Parameters were set as  $\alpha=3$  and  $\beta=0.5$ . In other words, a subband signal and chrominance signal are expressed within the range of 3bit and 4bit, respectively. Figure 8 (a) shows an original color image, and Fig.8 (b) shows a color-embedded gray image by the proposed algorithm. Almost the same gray image can be obtained. Figure 8 (c) shows a color image recovered from only a gray image in Fig.8 (b). Although, in the enlarged image, we can recognize texture patterns, color image can be recovered adequately with keeping the spatial resolution and pixel depth of the chrominance component.

In order to compare the recovery performance of chroma with conventional technique in Ref.[5], we used a color image shown in Fig.1 (a) for the next experiment. We set the parameter  $\alpha=0$  for evaluating the performance under the same condition between the proposed algorithm and Ref.[5]. Figure 9 (a) shows the result by using the method in Ref.[5] again. The chroma deteriorates can be

confirmed. We show the color recovery results by the proposed algorithm in Figs. 9 (b) and (c). We set the parameter to  $\beta=0.2$  and 0.5, respectively. As shown in Fig. 9, the chroma is improved remarkably by using the proposed algorithm.

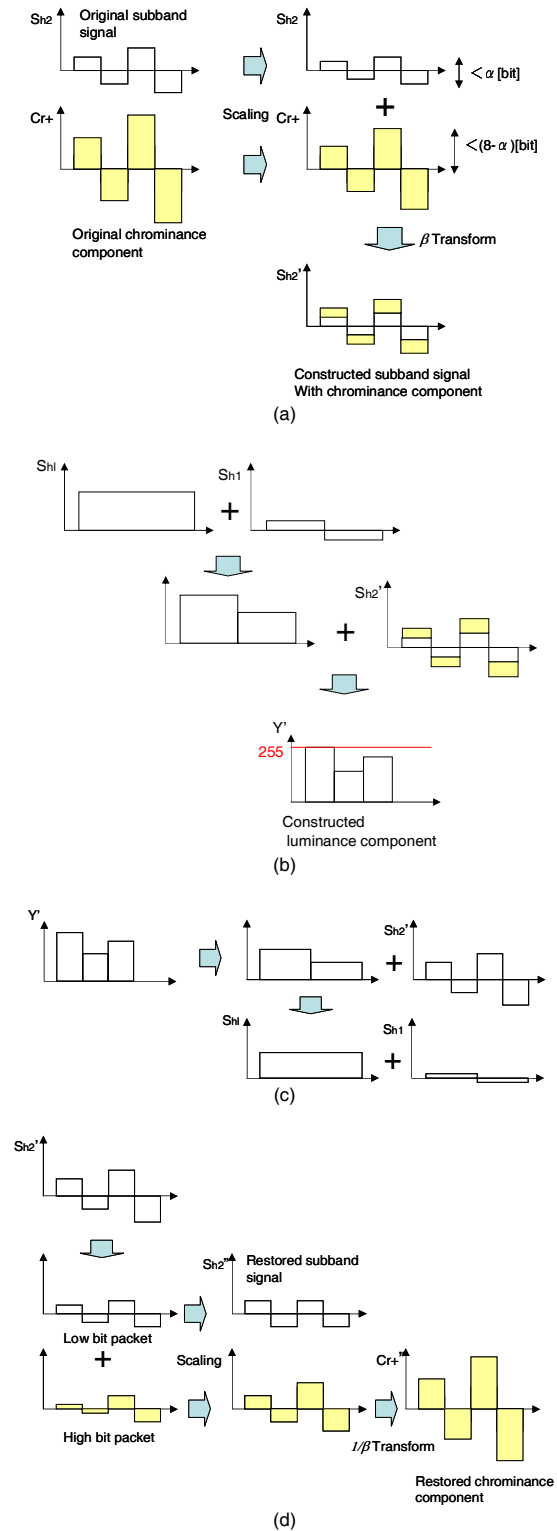


Figure 7 Signal processing with  $\beta$ -transform.



Figure 8 Color recovering result by the proposed algorithm

Next, we used a color image *Lena* (212x212) in Fig. 10 (a) which lost spatial resolution to compare the resolution with the conventional technique. Figure 10 (b) shows a zoom-up image of the original color image. Figure 10 (c) shows the color recovered image using Ref.[5]. Since high frequency components were thrown away, spatial resolution deteriorates. From Figs. 10 (d) to (f) show the color recovered results by the proposed algorithm in the case  $\beta=1$ . As increasing parameter  $\alpha$ , the spatial resolution is improved.

Figure 11 (a) shows standard color image “wool” (1024x768). The textured gray image is shown in Fig. 11 (b). The resulting image after color recovery is shown in Fig. 11 (c). The parameters are set as  $\alpha=3$  and  $\beta=0.9$ . Figure 12 shows the quality of recovered color for parameter  $\beta$  using CIELAB color difference  $\Delta E_{ab}^*$ . Close-up images are shown in Fig. 12. Figures 12 (a) and (b) show

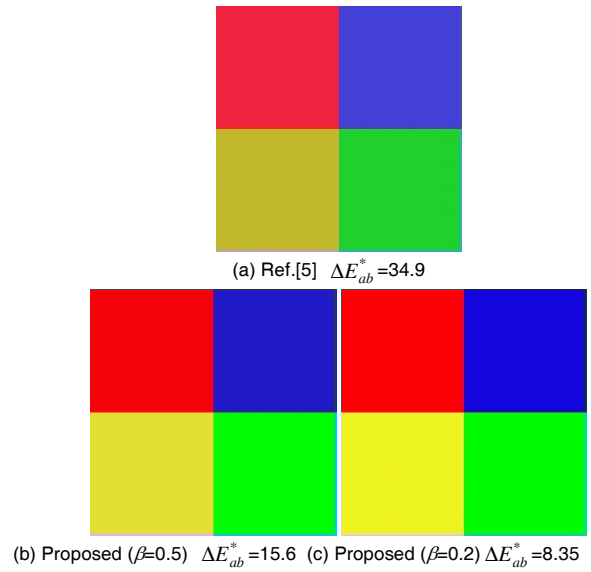


Figure 9 Comparison for chroma ( $\alpha=0$ )



Figure 10 Comparison for spatial resolution

close-up images of Figs. 11 (a) and (c), respectively. As shown in Fig.12 (b), the colors are a little de-saturated and some deterioration of the resolution and quantization can be recognized, but the result is almost good reproduction. Especially for resolution, we can recognize characters in the image.

## Conclusions

This paper has proposed a semi-invariant method for chroma and spatial resolution to convert color images to gray, and verified the effectiveness of the proposed algorithm through color recovery experiments. The proposed algorithm can recover color image from the gray image with keeping chroma and spatial resolution. There are two parameters in our algorithm. The parameter  $\alpha$  is used for controlling the trade-off between the spatial resolution and chrominance restoration. The parameter  $\beta$  on chrominance expression is used for controlling the recovered chroma and pixel depth. The method can be applied to various fields of imaging technology. The practical application and improvement of color recovery precision are future problems.

## References

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## Author Biography

Takahiko Horiuchi received his B.E., M.E. and Ph.D. degrees from University of Tsukuba in 1990, 1993 and 1995, respectively. He was a member of the Promotion of Science for Japanese Junior Scientists from 1993 to 1995. From 1995 to 1998, he was an Assistant Professor with the Institute of Information Sciences and Electronics, University of Tsukuba. From 1998 to 2003, he was an Associate Professor with the Faculty of Software and Information Sciences, Iwate Prefectural University. In 2003, he moved to Chiba University. He is an Associate Professor at Graduate School of Advanced Integration Science



(a) Original image



(b) Constructed gray image



(c) Recovered color image by the proposed algorithm

Figure 11 Color recovery results for "wool" image.



(a) Close-up of the original image in Fig.11(a)



(b) Close-up of the image by the proposed algorithm in Fig.11(c)

Figure 12 Close-up of the experimental results in Fig.11.